

# Operational Stacks and Reverse Polish Notation

In 1951, Jan Lukasiewicz' book on formal logic first demonstrated that arbitrary expressions could be specified unambiguously without parentheses by placing operators immediately before or after their operands. For example, the expression

$$(a + b) \times (c - d)$$

is specified in operator prefix notation as

$$\times + ab - cd$$

which may be read as multiply the sum a plus b by the difference c minus d. Similarly, the expression can be specified in operator postfix notation as

$$ab + cd - \times$$

with the same meaning. In honor of Lukasiewicz, prefix and postfix notation became widely known as Polish and reverse Polish, respectively.

During the following decade the merits of reverse Polish notation were studied and two simplifications in the execution of computer arithmetic were discovered. First, as reverse Polish notation is scanned from left to right, every operator that is encountered may be executed immediately. This is in contrast to notation with parentheses where the execution of operators must be delayed. In the above example,  $(a + b) \times (c - d)$ , the multiply must wait until  $(c - d)$  is evaluated. This requires additional memory and bookkeeping. Second, if a stack (that is, a last-in-first-out memory) is used to store operands as a reverse Polish expression is evaluated, the operands that an operator requires are always at the bottom of the stack (last operands entered). For  $(a + b) \times (c - d)$ , the reverse Polish,  $ab + cd - \times$ , is evaluated as follows.

	T							
	Z					a + b		
	Y		a		a + b	c	a + b	
↑ stack	X	a	b	a + b	c	d	c - d	(a + b) X (c - d)
Reverse Polish		Enter a	Enter b	+	Enter c	Enter d	-	X

These properties have made the notation a valuable tool in the computer industry. All modern computer compilers for languages such as FORTRAN and ALGOL convert statements to reverse Polish in some form before producing a program that can be executed. Some computer manufacturers have even designed their machines with special instructions to perform stack operations to facilitate execution of reverse Polish. However, the HP-35 is the first scientific calculator to fully exploit the advantages of reverse Polish and automatic stack operations to provide user convenience seldom found in calculators.